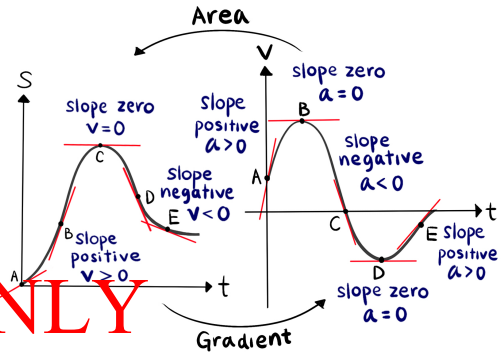
Kinematics

Equations of Motion: For a particle moving under constant acceleration, the displacement, velocity, acceleration and time are related by the 3 equations of motion.

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

Kinematic Graphs: Since displacement, velocity and acceleration are related through the time derivative, the velocity-time and acceleration-time graphs can be obtained by finding the gradients of the displacement-time and velocity-time graphs respectively.
Likewise, the displacement-time and velocity-time graphs can be obtained by finding the area under the velocity-time and acceleration-time graphs respectively.

$$v = \frac{ds}{dt},\ a = \frac{dv}{dt}.$$
$$s = \int v dt,\ v = \int a dt.$$



Projectile Motion: Particles launched at an angle oblique to the earth's gravitational field will follow a curved path, known as projectile motion.

Without air resistance, the particle experiences zero acceleration in the horizontal direction and acceleration of $g = 9.81\text{ms}^{-2}$ in the vertical direction.

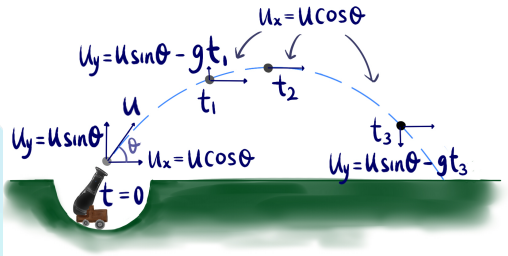
The motion can be analysed by decomposing all vector components vertically and horizontally.

x-direction $(\rightarrow^+)$

$$a_x = 0,\ v_x = u_x$$
$$s_x = u_x t$$

y-direction $(\uparrow^+)$

$$a_y = g \text{ (constant)}$$
$$v_y = u_y - gt$$
$$s_y = u_y t - \frac{1}{2}gt^2$$
$$v_y^2 = u_y^2 - 2gs_y$$



Common Calculations for Projectile Motions

Max Height $h_{\max}$

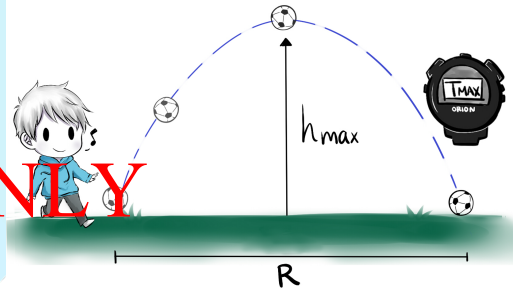
$$v_y^2 = u_y^2 + 2as_y$$
$$0 = u_y^2 - 2gs_y$$
$$s_y = u_y^2 / 2g$$
$$h_{\max} = (u\sin\theta)^2 / 2g$$

Time for Max Range

$$s_y = u_y t - \frac{1}{2}gt^2$$
$$0 = u_y t_{\max} - \frac{1}{2}gt_{\max}^2$$
$$t_{\max} = 2u_y / g = 2u\sin\theta / g$$

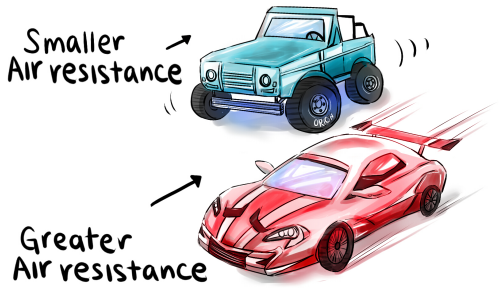
Range R

$$s_x = u_x t = u\cos\theta t$$
$$s_x = u\cos\theta(2u\sin\theta / g)$$
$$s_x = u^2 \sin 2\theta / g$$
$$R = u^2 \sin 2\theta / g$$
$$R_{\max} = u^2 / g$$

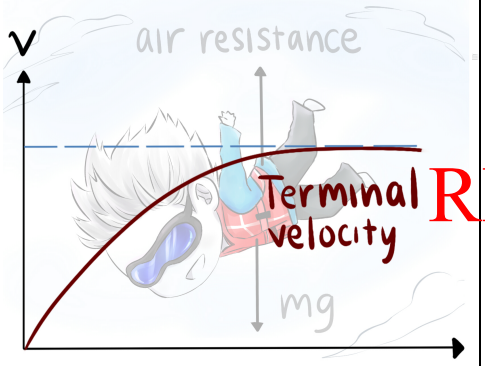


Air Resistance: In real life situation, particles moving through air will experience a resistive force F_D, called air resistance, which acts in the opposite direction of motion.

$F_D \propto v$ at low speeds
$F_D \propto v^2$ at high speeds



Effect of Air Resistance on **Free Falls**: Particle acceleration decreases until zero, when resistive forces equal weight. As acceleration goes to zero, it will then reach terminal velocity.



Effect of Air Resistance on **Projectile Motions**

- ✓ Particle will reach a lower maximum height h_2 as air resistance is in the same direction as gravity, which increases the magnitude of deceleration on its way up.
- ✓ Time taken to travel up t_1 is shorter than time taken to travel down t_2 as the magnitude of deceleration is greater than the magnitude of acceleration.
- ✓ Particle will travel a shorter range as air resistance always acts to slow down the horizontal velocity, which results in a smaller distance travelled.