1. Inequalities

Do **not** use a graphic calculator in answering this question.
Find the range of values of x that satisfy the inequality

$$(9-x)(10+|x-1|)>11,$$

giving your answers in exact form. [5]

2. APGP

The bank offers Mary two plans: a savings plan and an investment plan. Both require a one-time, initial amount to be placed in the plan.

Savings plan	**Investment plan**
Fixed interest of 4% Yearly	For odd-numbered years (e.g. 2013): Interest of 5% per annum. For even-numbered years (e.g. 2014): Interest of 1% per annum

For both plan the interest is credited back to earn further interest. She begins saving at the start of 2013. Explain whether it is more profitable for her to place her money in the savings or investment plan if she saves for an even number of years. [5]

3. Sigma Notation

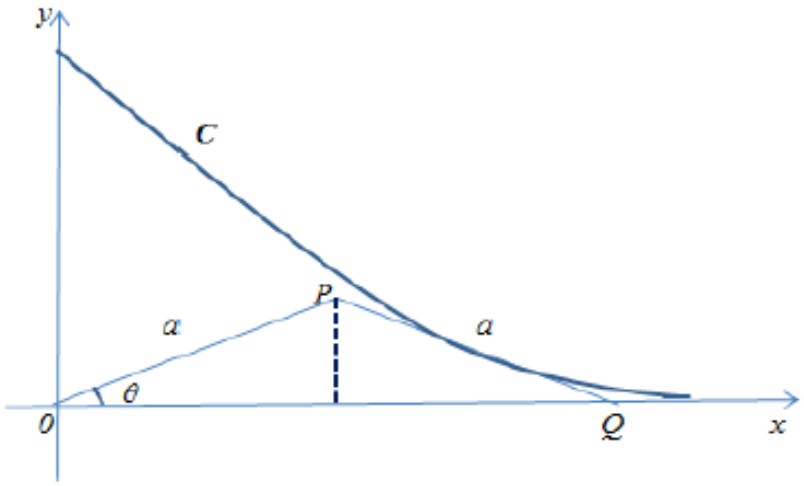
By using the identity $4\sin^3\theta \equiv 3\sin\theta - \sin 3\theta$, show that

$$\sum_{r=0}^{n}\frac{1}{3^r}\sin^3\left(3^r\theta\right)=\frac{1}{4}\left[3\sin\theta-\frac{1}{3^n}\sin\left(3^{n+1}\theta\right)\right]. \quad [3]$$

Hence, find the infinite sum of the following,

$$\sin^3\left(\frac{\pi}{2}\right)+\frac{1}{3}\sin^3\left(\frac{3\pi}{2}\right)+\frac{1}{3^2}\sin^3\left(\frac{3^2\pi}{2}\right)+\frac{1}{3^3}\sin^3\left(\frac{3^3\pi}{2}\right)+\cdots \quad [2]$$

4. Differentiation



In the diagram above, the curve C has parametric equation $x = 2\alpha\cos^3 t, y = 2\alpha\sin^3 t$, where $0 \le t \le \frac{\pi}{2}$. In the x-y plane, the origin is O and the variable point P lies in the first quadrant such that $OP = \alpha$ and OP makes an angle θ with the positive x-axis.

i. Find the equation of the tangent to C at the point where $t = \theta$. [4]

ii. Find the Cartesian equation of the locus of P. Hence, show that, as θ varies from 0 to $\frac{\pi}{2}$, the locus of P touches C at the point where $x = \frac{\pi}{4}$. [5]

The variable point Q is such that it lies on the positive x-axis and $OP = PQ = \alpha$.

iii. Find at the point where $\theta = \frac{\pi}{6}$, the rate of change of the area of triangle POQ when θ is increasing at a rate of 0.5 radian/sec. [4]