Surds and Indices

1.0 Introduction to Surds

It is an *nth* root of a positive integer, which cannot be expressed in exact form as a rational number. For example $\sqrt{4}=2$ and $\sqrt{\frac{4}{9}}=\frac{2}{3}$ are rational number but $\sqrt{6}$ is a surd. In this chapter we will be dealing mainly with surds involving square root.

1.1 Algebraic Manipulation

In general, we have the following rules:

$$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$$
$$\sqrt{a}\times\sqrt{b}=\sqrt{ab}$$
$$\sqrt{a}\times\sqrt{a}=a$$
$$m\sqrt{a}\pm n\sqrt{a}=(m\pm n)\sqrt{a}$$

Example 1

Which of the following is a surd ?

(a) $\sqrt{144}$ (b) $\sqrt{12}$ (c) $\sqrt{\frac{1}{16}}$ (d) $\sqrt{\frac{2}{16}}$ (e) $\sqrt[3]{64}$

Surds $\sqrt{k}$ is said to be in its most simplified form if k does not have any factor that is a perfect square (other than 1). How about $\sqrt[3]{J}$?

Example 2

Simplify each of the following

(a) $6\sqrt{5}-7\sqrt{125}$ (b) $\sqrt{8}+\sqrt{12}$ (c) $\sqrt{5}\times\sqrt{3}+\sqrt{60}$

(d) $\sqrt{28}+\sqrt{112}-\sqrt{252}$ (e) $\sqrt{48}\ x\ 8\sqrt{3}\div\sqrt{243}$

(f) $(2-\sqrt{3})(2+\sqrt{3})$ (g) $(5\sqrt{3}-3)^2$ (h) $(3\sqrt{2}-2)^3$

1.2 Rationalising Surds

The process of eliminating the surds in the denominator of a fraction is known as rationalizing.

For example, $\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$.

Use the following results for rationalization:

Product of conjugate pairs = rational nunber

i.e. $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})=a-b$

Example 3

Rationalise the denominator of the following fractions.

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{3}{1-\sqrt{5}}$ (c) $\frac{\sqrt{5}-2\sqrt{3}}{2\sqrt{5}-\sqrt{3}}$

Example 4

Simplify the following.

(a) $\frac{1}{\sqrt{2}}-\frac{2}{\sqrt{8}}+\frac{\sqrt{128}}{3}$

(b) $(\frac{1}{\sqrt{2}-1}-\sqrt{3}+2)\times2\sqrt{12}$

(c) $\left(\frac{\sqrt{2}}{3-\sqrt{6}}\right)^2$

Example 5

Simplify the following.

(a) $\frac{1}{\sqrt{2}}-\frac{2}{\sqrt{8}}+\frac{\sqrt{128}}{3}$

(b) $(\frac{1}{\sqrt{2}-1}-\sqrt{3}+2)\times2\sqrt{12}$

(c) $\left(\frac{\sqrt{2}}{3-\sqrt{6}}\right)^2$