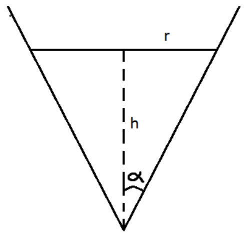
1.



$$\tan\alpha = \frac{r}{h} = 0.5$$
$$\Rightarrow r = 0.5h$$
$$v = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(0.5h)^2 h$$
$$= \frac{1}{12}\pi h^3$$
$$v = 3 \Rightarrow 3 = \frac{1}{12}\pi h^3$$
$$h = \sqrt[3]{\frac{36}{\pi}}$$
$$\frac{dv}{dh} = \frac{1}{4}\pi h^2$$
$$\frac{dh}{dt} = \frac{dv}{dt} / \frac{dv}{dh}$$
$$= \frac{0.1}{\frac{1}{4}\pi\left(\sqrt[3]{\frac{36}{\pi}}\right)^2}$$
$$= \frac{0.4}{\pi\left(\frac{36}{\pi}\right)^{\frac{2}{3}}}$$
$$= \frac{0.4}{\pi^{\frac{1}{3}}36^{\frac{2}{3}}}$$

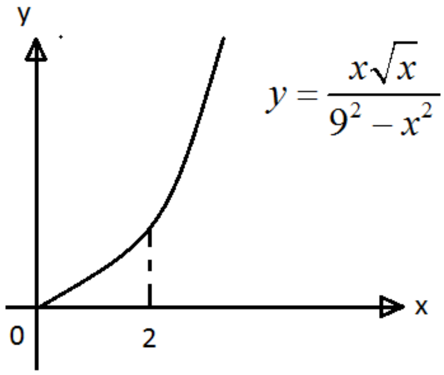
2. (a)(i)

$$\int x^2\cos nx\,dx = \left[x^2\times\frac{1}{n}\sin nx\right] - \int 2x\times\frac{1}{n}\sin nx\,dx$$
$$= \frac{x^2}{n}\sin(nx) - \frac{2}{n}\left[\left[x\times\left(\frac{-1}{n}\cos nx\right)\right] - \int -\frac{1}{n}\cos nx\,dx\right]$$
$$= \frac{x^2}{n}\sin(nx) - \frac{2}{n}\left[-\frac{x}{n}\cos(nx) + \frac{1}{n^2}\sin nx\right] + c$$
$$= \frac{x^2}{n}\sin(nx) + \frac{2x}{n^2}\cos(nx) - \frac{2}{n^3}\sin(nx) + c$$

(ii)

$$\int_{\pi}^{2\pi} x^2\cos nx\,dx = \left[\frac{x^2}{n}\sin(nx) + \frac{2x}{n^2}\cos(nx) - \frac{2}{n^3}\sin(nx)\right]_{\pi}^{2\pi}$$
$$= \left[\left(0 + \frac{4\pi}{n^2} - 0\right) - \left(0 + \frac{2\pi}{n^2}(-1)^n - 0\right)\right]$$
$$= \frac{4\pi}{n^2} - \frac{2\pi}{n^2}(-1)^n$$
$$= \frac{\pi}{n^2}\left(4 - 2(-1)^n\right)$$
$$\Rightarrow a = 4 - 1 \text{ or } 4 - (-2)$$
$$= 2 \text{ or } 6$$

(b)



$$V=\int_0^2 \pi y^2 dx$$

$$=\pi\int_0^2 \frac{x^2\times x}{\left(9-x^2\right)^2}dx$$

$$=\pi\int_9^5 \frac{x^3}{u^2}\times\left(-\frac{1}{2x}\right)du$$

$$=-\pi\int_9^5 \frac{9-u}{u^2}du$$

$$=-\pi\int_9^5 \frac{9}{u^2}-\frac{1}{u}du$$

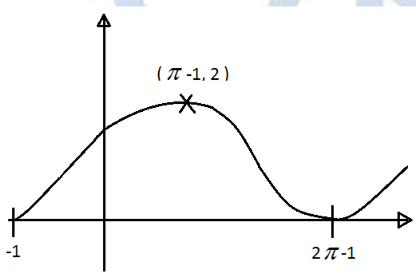
$$=-\pi\left[-\frac{9}{u}-\ln|u|\right]_9^5$$

$$=-\pi\left[-\frac{9}{5}-\ln 5-\left(-\frac{9}{9}-\ln 9\right)\right]$$

$$=-\pi\left[-\frac{4}{5}+\ln\left(\frac{9}{5}\right)\right]$$

$$=\pi\left[\frac{4}{5}-\ln\left(\frac{9}{5}\right)\right]$$

3. (i)



$$\frac{dy}{dx}=\frac{\sin t}{1+\sin t}=0$$
$$t=0,\pi,2\pi$$

(ii) $$\int_0^{t=a} y\,dx=\int(1-\cos t)(1+\sin t)\,dt$$

$$=\int 1+\sin t-\cos t-\sin t\cos t\,dt$$

$$=\int 1+\sin t-\cos t-\frac{1}{2}\sin 2t\,dt$$

$$=\left[t-\cos t-\sin t+\frac{1}{4}\cos 2t\right]_0^9$$

$$=a-\cos a-\sin a+\frac{1}{4}\cos 2a+1-\frac{1}{4}$$

$$=a+\frac{3}{4}-\sin a-\cos a+\frac{1}{4}\cos 2a$$

(iii) Normal equation:

$$\frac{y-\left(1-\cos\frac{\pi}{2}\right)}{x-\left(\frac{\pi}{2}-\cos\frac{\pi}{2}\right)}=2$$

$$\frac{y-1}{x-\frac{\pi}{2}}=2$$

$$y=2x-\pi+1$$

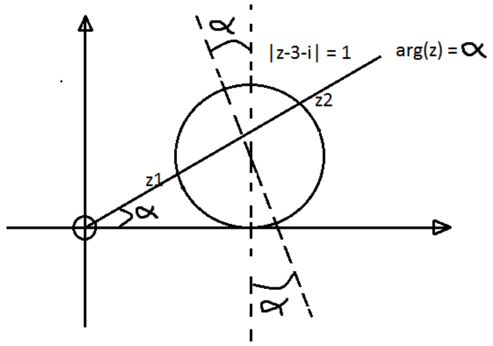
$$x=0,\ y=\pi-1$$

$$y=0,\ x=\frac{\pi-1}{2}$$

$$\text{area}=\frac{1}{2}\left(\frac{\pi-1}{2}\right)(\pi-1)$$

$$=\frac{1}{4}(\pi-1)^2$$

4. (i)



(ii) The perpendicular bisector intersects the center of the circle.
Therefore, the 2 points are:

$\left(3-1\sin\alpha\right)+i\left(1+1\cos\alpha\right)$

and

$\left(3+1\sin\alpha\right)+i\left(1-1\cos\alpha\right)$

$\Rightarrow\left(3\mp\frac{2}{\sqrt{29}}\right)+i\left(1\pm\frac{5}{\sqrt{29}}\right)$

(bi) $2-2i=2\sqrt{2}e^{i\left(-\frac{\pi}{4}\right)}$

$w=\left(2\sqrt{2}\right)^{\frac{1}{3}}e^{i\left(-\frac{\pi}{4+3}+\frac{2\pi k}{3}\right)}$

$=\sqrt{2}e^{i\left(-\frac{\pi}{12}+\frac{2}{3}\pi k\right)}$

$=\sqrt{2}e^{i\left(-\frac{\pi}{12}\right)},\sqrt{2}e^{i\left(\frac{7}{12}\pi\right)},\sqrt{2}e^{i\left(-\frac{3\pi}{4}\right)}$

(bii) $\arg\left(w^{*}w^{n}\right)=\frac{1}{2}\pi$

$\arg w^{*}+n\arg w=\frac{1}{2}\pi$

$=\left(4k+1\right)\frac{\pi}{2}$

$\frac{\pi}{4}+n\left(-\frac{\pi}{4}\right)=\frac{1}{2}\pi$

$=\left(4k+1\right)\frac{\pi}{2}$

$-\frac{\pi}{4}n=2\pi k+\frac{\pi}{2}-\frac{\pi}{4}$

$-\frac{\pi}{4}n=2\pi k+\frac{1}{4}\pi$

$n=-8k-1$

Least +ve $n=7$

5. (i) $\frac{4}{7}\times\frac{1}{6}+\frac{2}{7}\times\frac{2}{6}+\frac{1}{7}\times\frac{3}{6}=\frac{11}{42}$

(ii) p (B/player win) = $\frac{2}{7}\times\frac{2}{6}=\frac{4}{11}$

(iii) $\left(\frac{4}{7}\times\frac{1}{6}\right)\times\left(\frac{2}{7}\times\frac{2}{6}\right)\times\left(\frac{1}{7}\times\frac{3}{6}\right)\times 3!=\frac{4}{1029}$

6. (iii) $n=80$

$\mu_o=37$

$\sigma^2=140$

$H_o:\mu_o=37$

$H_1:\mu_o<37$

$z=\frac{\bar{x}-37}{\sqrt{\frac{140}{80}}}<-1.645$

$\bar{x}<34.823$

$\bar{x}<34.8$

(iv) $z=\frac{35.2-37}{\sqrt{\frac{140}{80}}}=-1.36067$

$p\left(z<-1.3607\right)=0.0868$

% significant level $<8.68\%$

7. (i) ${}^{4}C_3\times 3!=24$

(ii) $\left({}^{10}C_3-{}^{6}C_3-4C_3\right)\times 3!=576$

(iii) $\frac{(8-1)!\times 3!}{(10-1)!}=\frac{30240}{9!}=\frac{1}{12}$

(iv) Total – all together – 2 together =

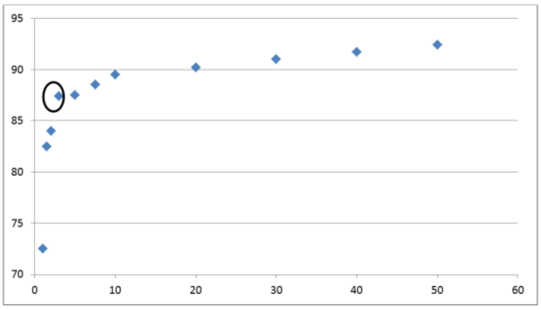
$\frac{(10-1)!-30240-\left({}^{3}C_2\times{}^{7}C_2\times(7-1)!\times 2!\times 2!\right)}{(10-1)!}$

$=\frac{(10-1)!-30240-181400}{(10-1)!}$

$=\frac{151200}{(10-1)!}$

$=\frac{5}{12}$

8. (i) The wrong point is (3,87.4)



(ii) From the scatter diagram, the value of y increases at a decreasing rate, hence a linear relationship is not suitable.

(iii) c is negative because y increase with x, which implies that y decreases with (1/x), therefore c must be negative. d is positive because horizontal asymptote of data points is positive.

(iv) $r = 0.980$

(v) Using the model, the regression line of y on x is $y = 91.75 - \dfrac{17.48}{x}$.

When x=3, y=85.9.

This estimate is reliable because

1) x=3 is within data range, i.e. interpolation was used.
2) The value of r indicates strong linear correlation between y and 1/x.

9. (a) $X \sim N\left(15, a^2\right)$

$P(10 < X < 20) = 0.5$

$P\left(-\dfrac{5}{a} < Z < \dfrac{5}{a}\right) = 0.5$

$P\left(|Z| < \dfrac{5}{a}\right) = 0.5$

$2P\left(Z < \dfrac{5}{a}\right) - 1 = 0.5$

$P\left(Z < \dfrac{5}{a}\right) = 0.75$

$\dfrac{5}{a} = 0.675$

$\therefore a = 7.41$

(b) $Y \sim B(4, p)$

$P(Y=1) + P(Y=2) = 0.5$

$\binom{4}{1}\left(p^1\right)(1-p)^3 + \binom{4}{2}\left(p^2\right)(1-p)^2 = 0.5$

$4p\left(1-3p+3p^2-p^3\right) + 6p^2\left(1-2p+p^2\right) = 0.5$

$2p^4 - 6p^2 + 4p = 0.5$

$\therefore 4p^4 - 12p^2 + 8p = 1$ (shown)

Using GC,

$p = -2.01(\text{rej}), 1.28(\text{rej}), 0.599 \text{ or } 0.166$

(c) $X \sim B\left(100, \dfrac{1}{3}\right)$

$n = 100 > 50, np = 33.3 > 5, nq = 66.7 > 5$

$\Rightarrow$ X can be approximated to a normal distribution.

$X \sim N\left(\dfrac{100}{3}, \dfrac{200}{9}\right)$ approx

$\mathrm{P}(X \ge 30) \xrightarrow{CC} P(X \ge 29.5) = 0.792$

10. (i) Average number of weeds is constant for different parts of the field with the same area.

Number of weeds in 1 part of the field is independent of number of weeds in the other parts of the field.

(ii) Let X : number of dandelion weeds in 1m^2

$X \sim Po(1.5)$

$P(X \ge 2) = 1 - P(X \le 1)$

$= 0.442$

(iii) $4X \sim Po(6)$

$P(4X \le 3) = 0.151$

(iv) $80X \sim Po(120)$

Since λ>10,

80X can be approximated using a normal distribution.

$80X \sim N(120, 120)$ approx

$\mathrm{P}(110 \le 80X \le 140) \xrightarrow{CC} \mathrm{P}(109.5 \le 80X \le 140.5)$

$= 0.800$

(v) Let Y: number of daisies in 1m^2

$Y \sim Po(\lambda)$

$P(Y \le 2) = P(2Y > 2)$

$P(Y \le 2) = 1 - P(2Y \le 2)$

Using GC to solve,

λ=1.85